

Approach to Modeling Continuous Turbine Engine Operation from Startup to Shutdown

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A generalized turbine engine start simulation (mathematical model) has been developed and demonstrated. The model, designated as ATEST-V3, is capable of simulating engine operation continuously from near static (zero speed) conditions to maximum engine power including windmill starting, spooldown starting, and starter-assisted starting. The enhanced capability to simulate the engine starting process provides the means to characterize and understand engine system operational behavior during critical startup and shutdown operations. ATEST-V3 is based on an aerothermodynamic matching of the major components. The component-matching technique is widely used for steady-state and transient turbine engine simulations that typically exclude subidle and starting operations. The same approach is shown to be applicable to engine starting operations by modeling component behavior continuously from zero to maximum power. The combination of an existing transient engine simulation and a numerically stable component-matching algorithm provided a foundation for extending the simulation capability to subidle engine operation and engine starting. ATEST-V3 was applied to a modern flight-type turbofan engine which demonstrated the capability to simulate windmill, spooldown, and starter-assisted starts at various flight conditions. Finally, a comparison is made between model results and engine test data.

Nomenclature

N = rotor speed, rpm
 NC = corrected rotor speed, rpm
 P = pressure, psia
 T = temperature, °R
 W = gas flow rate, lbm/s

Subscripts

HP = high-pressure rotor
LP = low-pressure rotor
 s = static condition
 t = total condition
0–9 = engine station designation

Introduction

SUCCESSFUL and quick starting of turbine engines is essential for safe flight operations of an aircraft and activation of reserve power-generating capacity of ground-based systems. A simulation of the starting process can provide valuable information such as the torque required to accelerate and the time required to start over a wide variety of starting conditions. In addition, a component-level aerothermodynamic simulation of the starting process can provide an indication of compression system stall margin and turbine temperature margin during an engine start, and an indication of

the ability of the engine control to execute and monitor a successful engine start.

Turbine engine simulation capabilities have evolved from models of steady-state engine operation in the early 1960s¹ to models of transient engine operation with control simulations in the early 1980s.² Early transient simulations^{3–9} were applicable to above-idle transient engine operations for a limited range of specific engine configurations (e.g., two-spool turbofan). More recently, transient simulations^{2,10,11} were generalized and became applicable to arbitrary engine configurations, but remained limited to above-idle transient engine operations. Further advancements provided the additional capability of simulating engine shutdowns.¹²

Several simplified component-level engine start models capable of simulating engine windmill and starter-assisted starts were introduced in the 1980s.^{13–15} The simplifications were based on subidle steady-state operating line relationships and avoided the requirements for detailed component performance maps. The steady-state relationships facilitated the simulation of the starting process but compromised the ability to simulate above-idle transient engine operation accurately. However, the implementation of the simplified start models demonstrated the ability to apply conventional component-matching principles to the starting process.

A generalized approach (i.e., applicable to arbitrary engine configurations) to modeling turbine engine operation continuously from startup to shutdown is described in this article. The approach preserves existing steady-state and transient capabilities and simulation accuracy for above-idle operations. This article will focus on the approach used in applying conventional component-matching simulation principles to the starting process. The approach is applied to a military flight-type two-spool afterburning turbofan engine, and simulation results are compared to engine test data.

Approach

General

An existing engine simulation technique (ATEST)¹⁶ was selected as a basis in providing an engine starting simulation

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because of its overall capabilities. The ATEST program provides a one-dimensional component-level transient simulation (0- to 20-Hz frequency response) applicable to arbitrary engine configurations. The ATEST program is also capable of simulating off-operating line engine operations and utilizes widely accepted component-matching principles. The ATEST program includes the effects of rotor dynamics, volume dynamics, and heat transfer, and it is capable of accepting an engine control simulation as an additional program module.

Fundamentally, engine windmilling is a steady-state process and engine starting is a transient process subject to the same aerothermodynamic principles as above-idle engine operation. The existing ATEST program provided the capability of simulating normal above-idle engine steady-state and transient operations. An expansion of ATEST to include the simulation of engine starting and unfired (no combustion) windmilling processes provides the capability to simulate continuous engine operation from startup to maximum power to shutdown with a single technique (Fig. 1).

Component Models

The expansion of ATEST to include the simulation of windmilling and start operations required expansion of the individual component (fan, compressor, burner, turbine, etc.) models. First, the compressor and turbine component models were expanded to provide simulation of compression, expansion, and frictional processes within each model. Next, the component model for the burner was expanded to include combustor light-off and blowout capability. The remaining duct, nozzle, and mixing component models were expanded to simulate continuous operation from zero to maximum gas flow conditions. Finally, the component models were assembled to provide a complete engine simulation which was designated as the advanced turbine engine simulation technique, version 3.0 (ATEST-V3). The details of expanding the component models are discussed in the following paragraphs.

Pumping capacity for the compressor and turbine models is defined by the classical relationship between pressure ratio (or expansion ratio) and corrected gas flow along lines of constant speed (Figs. 2 and 3). Compressor and turbine efficiency are defined by the relationship between temperature ratio and pressure ratio along lines of constant speed (Figs. 4 and 5). The pumping capacity and temperature ratio relationships are used to define component performance because each relationship is continuous across compression, expansion, and frictional processes (see Figs. 4 and 5).

Typically, the relationships defining component pumping capacity and efficiency are well-defined for above-idle engine operation (Figs. 2-5). The relationships were expanded to include zero-speed, zero-flow, expansion, compression, and frictional operating conditions (Figs. 2-5). Specifically, a streamline curvature program¹⁷ was used to estimate compressor performance for low (20-50%) compressor speeds. A zero-speed relationship for the compressor and turbine was

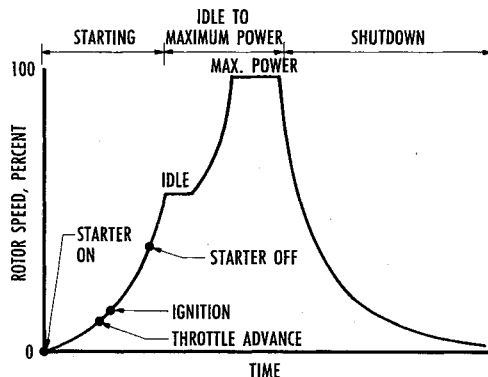


Fig. 1 Continuous simulation capability for engine operations from startup to maximum power.

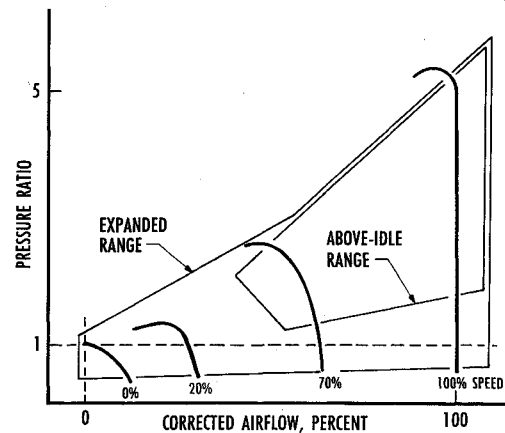


Fig. 2 Compressor pumping capacity; pressure ratio as a function of corrected airflow along lines of constant corrected speed.

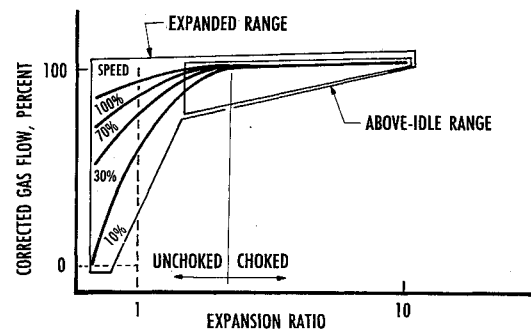


Fig. 3 Turbine pumping capacity; corrected gas flow as a function of expansion ratio along lines of constant corrected speed.

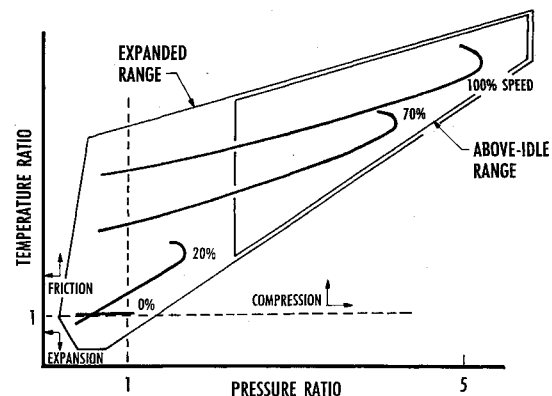


Fig. 4 Compressor temperature ratio as a function of pressure ratio along lines of constant corrected speed.

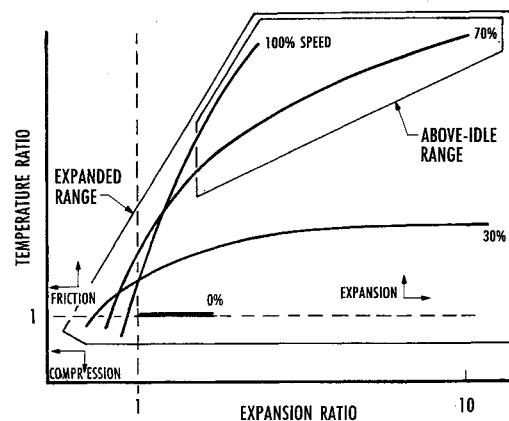


Fig. 5 Turbine temperature ratio as a function of expansion ratio along lines of constant corrected speed.

synthesized based on the zero-flow condition that exists at pressure ratio (PR) equal to unity and a friction loss that would exist when forcing air through the components with a seized rotor. The windmilling portion ($PR < 1$) of the compressor relationships and the unchoked low-speed portion of the turbine relationships were extrapolated from the above-idle relationships.

Component-matching principles rely on the pumping capability relationships to determine an operating point for each component. Generally, for above-idle operation, the determination of a turbine operating point is simplified because the turbine is choked and gas flow is insensitive to expansion ratio (Fig. 3). However, for subidle unchoked operation, corrected gas flow is highly sensitive to changes in expansion ratio. A similar phenomenon occurs in the exhaust nozzle. The large changes in sensitivity, rather than the magnitude of the sensitivity, during unchoked operation complicate the determination of turbine and nozzle operating points.

Separate techniques were employed within the turbine and nozzle models to facilitate the determination of their respective operating points during above-idle choked and subidle unchoked operations. The turbine model takes advantage of the low sensitivity of gas flow to expansion ratio during choked operation and the similarly low sensitivity of expansion ratio to gas flow during unchoked operation. Turbine gas flow is specified as a function of expansion ratio during choked turbine operations; conversely, turbine expansion ratio is specified as a function of gas flow for unchoked operations (Fig. 6). The relationship between gas flow and expansion ratio is equivalent for both choked and unchoked operations, ensuring a smooth transition between choked and unchoked operation. For the exhaust nozzle model, a flow parameter was defined in terms of static pressure (static flow parameter) rather than total pressure (total flow parameter). The changes in sensitivity of the static flow parameter to expansion ratio are more gradual than for the total flow parameter (Fig. 7). The more gradual changes in sensitivity facilitate the determination of the nozzle operating point over the entire nozzle operating regime.

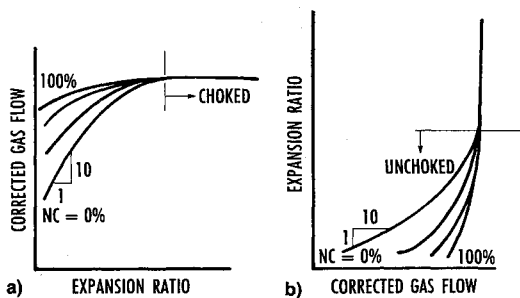


Fig. 6 Functional relationship between turbine gas flow and expansion ratio: a) choked turbine, b) unchoked turbine.

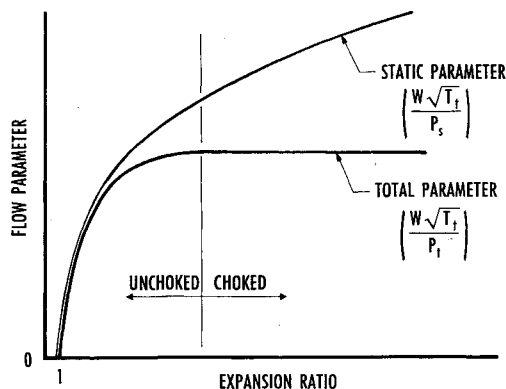


Fig. 7 Exhaust nozzle total and static flow parameter as a function of expansion ratio.

The main combustor model includes the capability to detect favorable ignition conditions and blowout (flame extinction) conditions. A combustor loading parameter similar to that proposed by Longwell¹⁸ is used to define a bounded region of stable combustion (Fig. 8). The combustor model inhibits heat release calculations when combustor flow conditions (fuel-air ratio, pressure, airflow, etc.) lie outside the bounded region. Ignition occurs when combustor flow conditions lie within the bounded region and an ignition source (e.g., electrical igniter) is present. Combustor blowout occurs when combustor flow conditions exit the stable combustion region. The stable combustion boundary is empirically determined and is unique for each combustor configuration.

The combustor stability boundary was generally defined based on combustor component testing. Calibration of the boundary for the specific application was based on a limited set of engine start-test data and included data only from the fuel-lean side of the stability boundary.

Component Matching Algorithm

The model of a turbine engine cannot be expressed analytically; therefore, an iterative approach is required to satisfy a set of implicit relationships that describe the engine. Rational, effective procedures and logic for the implementation of computerized analysis and simulation¹⁹ (REPLICAS®) is a collection of algorithms that provides reliable convergence of the implicit relationships. REPLICAS is based on a Newton-Raphson technique and is augmented by a matrix updating algorithm based on Broyden's method. REPLICAS is applicable to a wide variety of physical systems and is applied to both steady-state and transient engine simulation, including subidle and starting operations.

The Newton-Raphson technique requires the calculation of a Jacobian matrix that characterizes the engine. The elements of the Jacobian matrix commonly vary by several orders of magnitude, depending on the specific system being simulated. Large variations among the elements adversely affect the convergence reliability of the Newton-Raphson technique. REPLICAS uses a scaling algorithm for the Jacobian matrix and the Broyden update to the Jacobian matrix that eliminates large variations among the elements. The scaling algorithm improves convergence reliability, especially during the simulation of starting and shutdown processes as airflow and shaft speeds approach zero.

A set of iteration variables and corresponding physical laws establish the implicit relationships that describe the engine. Iteration variables include the rates of rotor acceleration and the individual operating points for the fan, compressor, high- and low-pressure turbines, and exhaust nozzle. The physical laws require that flow continuity be maintained at component interfaces and that nonzero net torque results in a corresponding rotor acceleration. Convergence is achieved by reducing the component flow mismatches and torque mismatches to zero (within a specified tolerance). REPLICAS provides a consistent normalized tolerance for each implicit

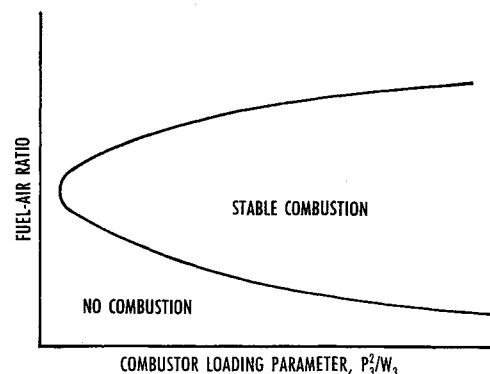


Fig. 8 Region of stable combustion.

relationship and uses both the differential and integral forms of Gear's second-order integration formula²⁰ to improve convergence reliability throughout the operational range of the engine.

The REPLICAS component matching algorithm determines the combination of component processes (expansion, compression, or frictional) that are required to achieve convergence. The reliable convergence properties of REPLICAS, combined with the expanded component capabilities to simulate the various processes provides the capability to simulate windmill, starting, subidle, above-idle, and shutdown operations.

Results

The ATEST-V3 model was applied to a military-type two-spool afterburning turbofan engine (Fig. 9) and included a simulation of the engine control. Model results were obtained by prescribing throttle position as a function of time and flight conditions in terms of altitude and Mach no. (K-ft/Mach). Model results were compared to engine test data for steady-state windmill operation and windmill, starter-assisted, and spooldown starts.

The ATEST-V3 model results were compared to engine test data to evaluate the capability of the model to simulate engine starting phenomena. The more important aspect of the evaluation was the ability of the model to reproduce the general shape of time-varying performance, including the existence of inflections, breakpoints, and overshoots. The comparison of absolute levels of performance was evaluated, but was less important in demonstrating the capability of simulating fundamental engine start processes.

The level of agreement discussed below was attained by expanding the component models and including the improved component matching algorithm as discussed previously. Further improvements to the level of agreement would result from revision of the control system simulation to match control system revisions specific to the tested engine and the inclusion of off-schedule geometry effects on component performance.

Windmilling and Windmill Start Simulation

During engine windmilling operation the fan and compressor behave as expansion, compression, or frictional devices. The turbines usually behave as an expansion or frictional device. The behavior of each component is determined by the component-matching algorithm and depends on the interaction among all the components and the engine ram

STATION NUMBER	STATION IDENTIFICATION
0	FREESTREAM
2	FAN INLET
3	COMPRESSOR EXIT
4	COMBUSTOR EXIT
5	TURBINE EXIT
7	EXHAUST NOZZLE INLET
8	EXHAUST NOZZLE THROAT
9	ENGINE EXIT

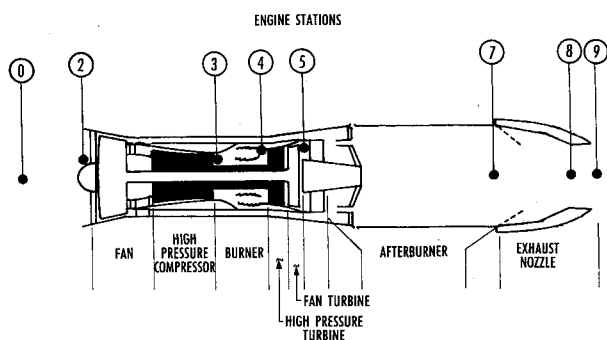


Fig. 9 Schematic diagram of two-spool afterburning turbofan engine.

ratio (defined as the ratio of engine inlet total pressure to exit static pressure).

Model results shown in Fig. 10 were obtained by simulating an engine shutdown at a specified ram ratio and allowing the rotor speeds to decelerate to a steady-state condition. The steady-state values of fan and compressor rotor speed are compared to engine test data for a range of engine ram ratio in Fig. 10. Generally, the test data show that rotor speed increases with increasing ram ratio. Two noticeable inflections occur in both the fan and compressor speed relationships to ram ratio. The general characteristic and the inflections in the engine speed relationships are also evident in the model results. Direct comparisons of the magnitude of speed at a specified ram ratio are better for fan speed than for compressor speed. Model results generally match test data to within 130 rpm for both rotors; however, the maximum discrepancy for compressor speed reached 340 rpm.

Model results for compressor speed and combustor pressure are compared to test data in Fig. 11 for a windmill engine start at a 5.5 K/0.8 (K-ft altitude/Mach no.) flight condition. Steady-state windmilling operation, the throttle advance from a cutoff to an idle setting, ignition, and idle operation are also indicated in Fig. 11. The test data indicate acceleration from a steady-state windmilling condition and a small overshoot (≈ 200 rpm) prior to stabilizing into a steady-state idle condition. An increasing rate of acceleration produces the curvature in the time-varying speed curve. A similar level of acceleration, the overshoot, and the curvature are also evident in the model results. The agreement for absolute levels of performance is also indicated in Fig. 11.

The agreement between model results and test data shown in Fig. 11 is representative of the agreement of other param-

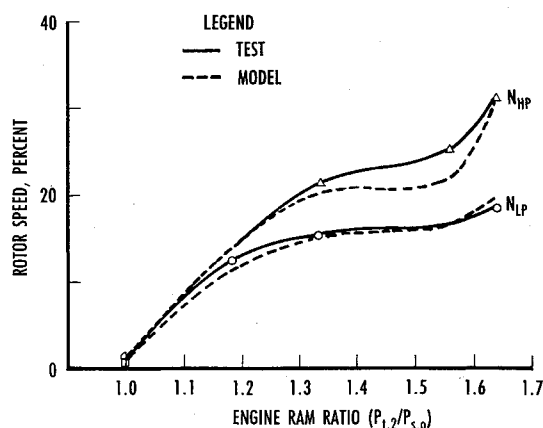


Fig. 10 Steady-state windmilling performance.

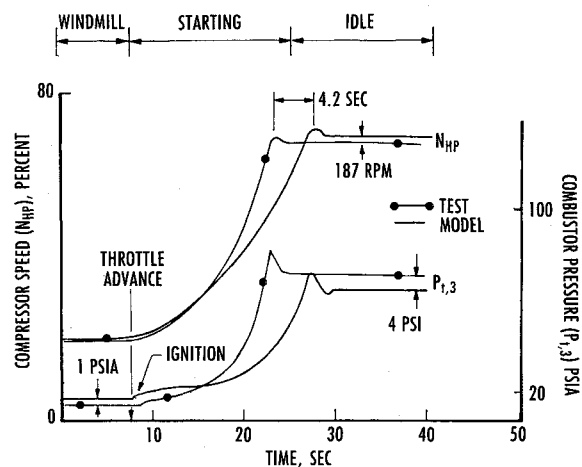


Fig. 11 Comparison of model results to engine test data for a windmill start (5.5 K/0.8).

eters (e.g., fan speed and fuel flow) at 5.5 K/0.8 and representative of model-to-test data agreement at other flight conditions.

Spooldown Start Simulation

Spooldown starts were accomplished by readvancing the throttle to idle as core speed decelerated to a specified speed (i.e., turnaround speed) in response to an engine shutdown. Typical turnaround speeds are 75, 60, 40, and 30% of the compressor speed at maximum power. In general, the turnaround speed is greater than the steady-state windmilling speed for the specific flight condition.

Model results for compressor speed and combustor pressure are compared to test data in Fig. 12 for a spooldown (35% turnaround speed) at a 25 K/0.6 flight condition. Intermediate power (maximum nonaugmented) operation, engine cutoff, throttle advance from cutoff to idle, ignition, and idle operation are also indicated. The test data indicate rapid deceleration (≈ 350 rpm/s) from intermediate power followed by ignition and acceleration prior to stabilizing at a steady-state idle condition. The rate of acceleration initially decreases and then increases, producing an inflection in the time-varying speed curve at approximately 52 s. The same level of deceleration and average acceleration is evident in the model results. The inflection is also present, but less obvious in the model results. The agreement for absolute levels of performance is also shown in Fig. 12.

The agreement shown between model and test data in Fig. 12 is representative of the agreement of other parameters (e.g., fan speed and fuel flow) for the 35% spooldown start at 25 K/0.6 and representative of agreement at other flight conditions and other turnaround speeds.

Starter-Assisted Start Simulation

An external power source is required for ground starting and for low values of ram ratio that alone do not produce sufficient aerothermodynamic conditions for starting. Consequently, a starter is required to assist in accelerating the engine core (high-pressure compressor and turbine) to self-sustaining speeds. A model of the engine starter provided the external power required to simulate starter-assisted starts.

Model results for fan and compressor speed are compared to test data in Fig. 13 for a starter-assisted start at a 10 K/0.5 flight condition. The steady-state windmilling condition, starter engagement, throttle advance from a cutoff to an idle setting, ignition, and idle operation are also indicated in Fig. 13. The test data indicate starter-induced acceleration from a steady-state windmill condition, an increased rate of acceleration after ignition, and a small overshoot (≈ 200 rpm) prior to stabilizing at a steady-state idle condition. Similar rates of

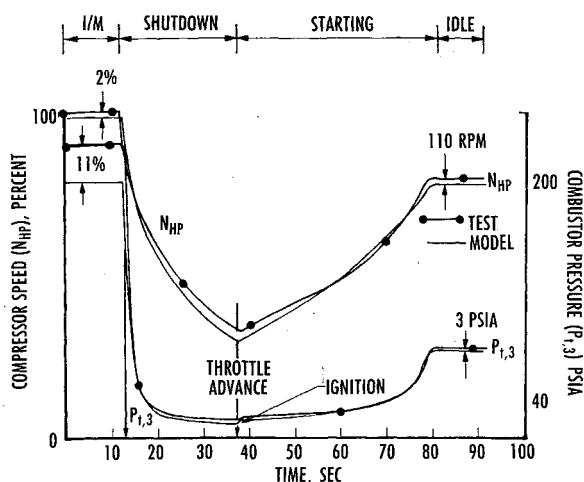


Fig. 12 Comparison of model results to engine test data for a 35% spooldown start (25 K/0.6).

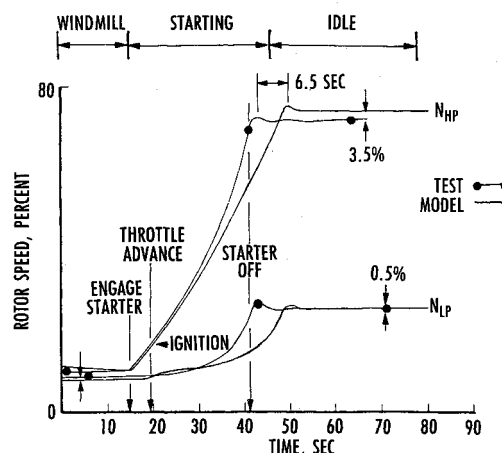


Fig. 13 Comparison of model results to engine test data for a starter-assisted start (10 K/0.5).

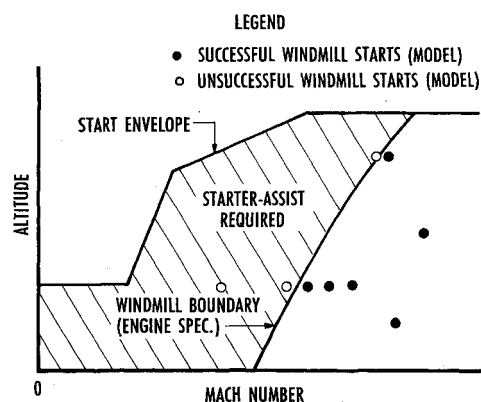


Fig. 14 ATEST-V3 model agreement with engine specification for windmill start boundary.

acceleration, the overshoot, and the curvature of the time-varying speed curves are also evident in the model results. The agreement for absolute levels of performance is also shown in Fig. 13.

The test results indicate an increasing rate of fan acceleration during the starting process. However, the model results are interrupted by a 5-s period of reduced fan acceleration. The reduced fan acceleration calculated by the model appears as an inflection in fan speed approximately 4 s after the throttle advance (Fig. 13). This phenomenon consistently appears in the model results during the transition from an expansion process to a compression process.

The phenomenon is attributed to a simplified simulation of a complex windmilling process in which compression and expansion processes occur simultaneously in the fan hub region and tip region, respectively. The simplification contributes to the longer starting times calculated by the model for starter-assisted and windmill starts. Simulation of the transition phenomenon would be improved by modeling the fan hub and the fan tip as individual components.

The agreement between model results and test data shown in Fig. 13 is representative of the agreement of other parameters (e.g., combustor pressure and fuel flow) at 10 K/0.5 and representative of model-to-test data agreement at other flight conditions.

Prediction of Starting Boundaries

The starting envelope, as defined by the engine manufacturer's specification for the two-spool turbofan application, is shown in Fig. 14. The envelope, in terms of altitude and Mach number, is divided into regions in which the engine specification requires that the engine start from windmill conditions and in which a starter assist is required.

The boundary between starter-assisted and windmill starting capability was successfully predicted by the model at two altitudes and is indicated by successful and unsuccessful windmill starts that straddle the boundary (Fig. 14). The determination of unsuccessful starts was based on excessive (>60 s) starting time as prescribed by the engine specification.

Limitations

The simulation of combustor ignition and blowout is a key area of modeling continuous engine operation from startup to shutdown. Combustor ignition is a complex phenomenon and is not well understood in the industry. Simple one-dimensional correlations that form ignition boundaries are highly empirical and unique to specific combustor configurations. Additional development is required in this key area to provide a more accurate determination of combustor ignition characteristics.

The ATEST model described in this article is capable of simulating windmilling speeds to less than 10 rpm. However, a simulated start from static conditions (absolute zero speed) is complicated by the indeterminate nature of the relationship between torque and power when speed approaches zero. It is expected that the simulation of engine operations at zero and near zero speeds can be attained by representing component performance in terms of torque rather than temperature ratio, which is a form of specific power.

Summary

A new capability in turbine engine modeling, ATEST-V3, has been developed and demonstrated. The ATEST-V3 capability to simulate the engine start process provides the means to quantitatively understand the combined effects of operating line excursions, turbine temperature fluctuations, engine control operation, and component interactions, in addition to power and air extraction effects during the engine start process.

The ATEST-V3 modeling approach is applicable to arbitrary engine configurations and provides the capability to simulate turbine engine operation continuously from startup to shutdown. The approach expands widely accepted component-matching principles to simulate subidle, windmill, and engine-starting operations and preserves existing one-dimensional steady-state and transient capabilities and simulation accuracy for above-idle operations.

ATEST-V3 results were validated using test data for steady-state windmilling operation, windmill starts, spool-down starts, and starter-assisted starts. ATEST-V3 successfully simulated the processes required to characterize turbine engine starting in addition to predicting the boundary between starter-assisted and windmill starts as prescribed by the engine manufacturer's performance specification.

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